Appendix - B
AC -
Item No. -

# As per NEP 2020

SRDSP Mandal's

## SHRI PANCHAM KHEMRAJ MAHAVIDYALAYA (AUTONOMOUS), SAWANTWADI

**Affiliated to University of Mumbai**

**Title of the Program**

**U. G. Degree in Mathematics**

**Syllabus for**

**Semester V and VI**

**(With effect from the Academic year 2026-2027)**

**Ref: GR dated 20th April, 2023 for Credit structure of UG**

# University of Mumbai

**S. R. D. S. P. Mandal's**

**SHRI PANCHAM KHEMRAJ MAHAVIDYALAYA**
**SAWANTWADI**
**(An Autonomous College)**
DIST: SINDHUDURG- 416 510, MAHARASHTRA
**DEPARTMENT OF MATHEMATICS**
Syllabus

| Sr. No. | Heading | Particulars |
|---|---|---|
| 1. | Title of Program | B. A. / B. Sc. (Mathematics) |
| 2. | Exit degree | U. G. Degree in Mathematics |
| 3. | Eligibility for Admission | U. G. Diploma in Mathematics |
| 4. | Scheme of Examination | NEP<br>40% Internal<br>60% External, Semester End Examination<br>Individual Passing in Internal and External Examinations. |
| 5. | Standard of Passing | 40% |
| 6. | Credit structure | Attached herewith |
| 7. | Semester | V and VI |
| 8. | Program Academic Level | 5.5 |
| 9. | Pattern | Semester (60:40) |
| 10. | Status | New |
| 11. | To be implemented from Academic Year | 2026-2027 |

Sign of the BOS **Chairman**
Dr. V. P. Sonalkar
BOS of Mathematics

Sign of

Sign of

## Member of the Board of Studies of Mathematics

| Sr. No. | Name of the Faculty | Category | Designation |
|---|---|---|---|
| 01 | Dr. Vishwas Pandurang Sonalkar | 12.5 (1) | HoD/ Chairman |
| 02 | Miss. Tanvi Dilip Shinde | 12.5 (2) | Member |
| 03 | Miss. Gayatri Rajesh Awate | | Member |
| 04 | Dr. Kishor D. Kucche | 12.5 (3) | Member |
| 05 | Dr. Jervin Zen Lobo | | Member |
| 06 | Dr. Shridhar Krishna Pawar | 12.5 (4) | Member |
| 07 | Asst. Prof. Sagar Balavant Patil | 12.5 (5) | Member |
| 08 | Miss. Shreya Shripad Bhagwat | 12.5 (6) | Member |
| 09 | Dr. Subhash Ishwar Unhale | 12.5 (7) | Member |

# Preamble

## 1. Introduction:

Shri Pancham Khemraj Mahavidyalaya, Sawantwadi (Autonomous) believes in implementing several measures to bring equity, efficiency and excellence in higher education system in conformity to the guidelines laid down by the University Grants Commission (UGC). In order to achieve these goals, all efforts are made to ensure high standards of education by implementing several steps to enhance the teaching- learning process, examination and evaluation techniques and ensuring the all-round development oflearners.

The institute has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the Third year B. Sc. / B. A. in Mathematics from the academic year 2026-2027. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematic s to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai and teachers from other University has prepared the syllabus of T.Y.B. Sc./ B. A. Mathematics. The present syllabi of T. Y. B. Sc. / B. A. for Semester V have been designed as per NEP 2020 guidelines. Curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of T. Y. B. Sc. / B. A. would consist of two semesters and semester V would comprise of five Major courses and semester VI, seven major courses. These courses contain analysis, calculus, abstract algebra, metric spaces along with elective courses (Number theory and it's applications, graph theory) and practical courses based on them. These courses develop strong logical thinking of learner and all these are having various applications in many recent trends of science and technology and practical component provides learner with hands-on experience in applying the theoretical concepts learned in all above courses and develops computation skill of learner.

## 2. Aims and Objectives:

- Give the students a sufficient knowledge of fundamental principles, methods, and a clear perception of in numerous powers of mathematical ideas and tools and know how to use them by modelling, solving, and interpreting.

- Reflecting on the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.
- Enhancing students' overall development and to equip them with mathematical modelling abilities, problem solving skills, creative talent, and power of communication necessary for various kinds of employment.
- A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

## 3. Program Outcome:

1) Integral Calculus: This course enables students to understand and apply the concepts of double and triple integrals, line, surface and volume integrals, and vector calculus theorems. It develops the ability to use Green's, Stokes', and Gauss' divergence theorems to solve problems in mathematics, physics, and engineering, and to interpret results in real-world contexts.

2. Group Theory and Ring Theory: This course enables students to understand and apply fundamental concepts of group theory, including groups, subgroups, homomorphisms, and isomorphism theorems. It also develops the ability to analyze ring structures such as rings, ideals, integral domains, and fields while strengthening logical reasoning and proof-writing skills.

3. Topology of Metric Spaces: This course enables students to understand the fundamental concepts of metric spaces, including open and closed sets, convergence, continuity, and completeness. It develops the ability to analyze topological properties such as compactness and connectedness and to apply rigorous proof techniques in the study of abstract mathematical structures.

4. Basic Complex Analysis: This course enables students to understand the fundamental concepts of complex numbers and complex-valued functions, including analyticity and conformality. It develops the ability to apply Cauchy–Riemann equations, complex integration, and Cauchy's theorems to solve problems, while strengthening analytical thinking and mathematical reasoning skills.

5. Number Theory and Its Applications: This course enables students to understand fundamental concepts of number theory, including divisibility, prime numbers, congruences and arithmetic functions. It develops the ability to apply number-theoretic

techniques to solve problems and explore applications in cryptography, coding theory and related areas.

6. Graph Theory: This course enables students to understand the fundamental concepts of graph theory, including graphs, subgraphs, connectivity, trees, and planar graphs. It develops the ability to apply graph-theoretic methods to solve problems in networks, algorithms, and combinatorial structures while enhancing logical reasoning and problem-solving skills.

## Credit Structure for Semester V

| Semester | Course Code | Title of the Course | Category of Course | Number of Credits |
|---|---|---|---|---|
| V (Level 5.5) | USMTT301 | Integral Calculus | **Major** | **2** |
| | USMTT302 | Group Theory | **Major** | **2** |
| | USMTT303 | Topology of Metric Spaces - I | **Major** | **2** |
| | USMTP304 | Practical - 301 (Practical based on Integral Calculus and Group Theory) | **Major** | **2** |
| | USMTP305 | Practical - 302 (Practical Based on Integral Calculus and Topology of Metric Spaces - I) | **Major** | **2** |
| | USMTET306A | Graph Theory – I | **Elective** | **2** |
| | USMTET306B | Number Theory and Its applications - I | **Elective** | **2** |
| | USMTEP307A | Practical - 303 (Practical based on Graph Theory - I) | **Elective** | **2** |
| | USMTEP307B | Practical - 304 (Practical based on Number Theory and Its applications - I) | **Elective** | **2** |
| | USMTT308 | Ordinary Differential equations | **Minor** | **2** |
| | MTVS04 | Computational Machine Learning | **VSC** | **2** |
| | MTCEP-01 | Community engagement programme | **CEP** | **2** |

## Credit Structure for Semester VI

| Semester | Course Code | Title of the Course | Category of Course | Number of Credits |
|---|---|---|---|---|
| **VI (Level 5.5)** | USMTT309 | Introduction to Complex Analysis | **Major** | **2** |
| | USMTT310 | Ring Theory | **Major** | **2** |
| | USMTT311 | Topology of Metric Spaces - II | **Major** | **2** |
| | USMTT312 | Advanced Real Analysis | **Major** | **2** |
| | USMTP313 | Practical - 305 (Practical based on Introduction to Complex Analysis) | **Major** | **2** |
| | USMTP314 | Practical - 306 (Practical based on Ring Theory) | **Major** | **2** |
| | USMTP315 | Practical - 307 (Practical based on Topology of Metric Spaces – II and Advanced Real Analysis) | **Major** | **2** |
| | USMTET316A | Graph Theory -II | **Elective** | **2** |
| | USMTET316B | Number Theory and Its applications - II | **Elective** | **2** |
| | USMTEP317A | Practical - 308 (Practical based on Graph Theory - II) | **Elective** | **2** |
| | USMTEP317B | Practical - 309 (Practical based on Number Theory and Its applications - II) | **Elective** | **2** |
| | USMTOJ-1 | ON JOB TRAINING | **OJT** | **4** |

# SEMESTER - V

## STRUCTURE OF THE COURSE

| Semester | Course code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTT301 **(MAJOR)** | Integral calculus | 2 | Theory | 30 |

**Course Objectives:**

- To provide a solid understanding of fundamental principles and methods, equipping students with the skills to apply mathematical ideas and tools through modeling, solving, and interpretation.
- To illustrate the expansive nature of the subject by fostering the acquisition of essential mathematical tools for continued studies across various scientific fields.
- To foster students' comprehensive development by placing emphasis on problem-solving skills, nurturing creative talents, and enhancing communication abilities, all of which are vital for a range of employment opportunities.
- To ensure exposure to both global and local issues within the realm of Mathematical Sciences, allowing learners to explore diverse aspects of the discipline.

**Course Outcomes:** After completion of the course, students will be able to

- Understand and remember the fundamental concepts of multiple integrals, including double and triple integrals, Fubini's theorem, change of variables, and line and surface integrals along with vector differential operators such as gradient, curl, and divergence.
- Apply techniques of multiple integration using Cartesian, polar, cylindrical, and spherical coordinates and evaluate line and surface integrals using Green's, Stokes', and the Divergence theorems.
- Analyze vector fields, parametrized curves, and surfaces to determine properties such as conservativeness, compute surface areas, and study the behavior of integrals under transformations and parameter changes.
- Justify/ check the correctness of computed integrals, verify conditions for conservativeness of vector fields, and confirm the applicability of major theorems such as Green's, Stokes', and the Divergence Theorem in specific situations.
- Construct counter examples to illustrate situations where integrability fails, where vector fields do not satisfy conditions for being conservative.

| **MODULE I) MULTIPLE INTEGRALS 15 LECTURES** |
|---|
| (i) Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp: box), Geometric interpretation as area and volume.<br>(ii) Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Following basic properties of double and triple integrals proved using the Fubini's theorem like Integrability of the sums, scalar multiples, products, (under suitable conditions) quotients of integrable functions and formulae for the integrals of sums and scalar multiples of integrable functions.<br>(iii) Integrability of continuous functions. More generally, Integrability of functions with a "small" set of (Here, the notion of "small sets" should include finite unions of graphs of continuous functions.) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains.<br>(iv) Change of variables formula (Statement only).<br>(v) Polar, cylindrical and spherical coordinates, and integration using these coordinates.<br>(vi) Differentiation under the integral sign. |
| **MODULE II) LINE, SURFACE AND VOLUME INTEGRALS 15 LECTURES** |
| (i) Review of Scalar and Vector fields on $\mathbb{R}^n$, Vector Differential Operators, Gradient, Curl, Divergence.<br>(ii) Paths (parametrized curves) in $\mathbb{R}^n$ (emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$). Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths.<br>(iii) Definition of the line integral of a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path-additivity and behaviour under a change of parameters. Examples.<br>(iv) Line integrals of the gradient vector field. Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative. Greens Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.<br>(vi) Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces. Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface.<br>(vii) Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence. Stoke's Theorem (Without proof). Examples.<br>(viii) Gauss' Divergence Theorem. Examples. |

**Recommanded Books:**


1. T. M. Apostol, Calculus, Vol. 2, Second Edition., John Wiley, New York, 1969 (Section 1.1 to 11.8).
2. James Stewart, Calculus Early Transcendentals, 8th Edition, Cengage Learning, USA, 2014. (Section 16.5 to 16.9).
3. Marsden and Jerrold E. Tromba, Vector Calculus, Fourth Edition, W. H. Freeman and Co., New York, 1996 (Section 6.2 to 6.4).


**Reference Books:**


1. T. Apostol, Mathematical Analysis, Second Edition, Narosa, New Delhi. 1947.
2. R. Courant and F. John, Introduction to Calculus and Analysis, Vol. 2, Springer Verlag, New York, 1989.
3. W. Fleming, Functions of Several Variables, Second Edition, Springer Verlag, New York, 1977.
4. M. H. Protter and C. B. Morrey Jr., Intermediate Calculus, Second Edition, Springer Verlag, New York, 1995.
5. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, (ISE Reprint), Addison Wesley publishing company, New York, 1998.
6. D. V. Widder, Advanced Calculus, Second Edition, Dover publications, Inc, New York, 1989.

**STRUCTURE OF THE COURSE**

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTT302 **(MAJOR)** | Group Theory | 2 | Theory | 30 |

## Course Objectives:

- To understand the fundamental concepts and properties of groups, including subgroups, order, and symmetry groups.
- To explore advanced topics in group theory, such as cosets, normal subgroups, and quotient groups, along with their applications.
- To analyze the structure of groups through homomorphisms, isomorphisms, and Cayley's theorem for finite groups.
- To investigate direct products of groups and cyclic groups, including their properties, generators, and relationships with other groups.

## Course Outcomes: After completion of the course, students will be able to

- explain basic ideas of groups, subgroups, homomorphisms, isomorphisms, internal and external direct products, cyclic groups, and their properties in simple terms.
- use subgroup criteria to identify subgroups, compute orders of groups and elements, and determine generators for cyclic and finite groups.
- partition groups into cosets, apply Lagrange's theorem to infer possible orders of subgroups, and analyze normality of subgroups.
- compute kernels and images of homomorphisms; apply the Isomorphism Theorems and Cayley's theorem to identify isomorphic structures.
- determine when direct products of cyclic groups are cyclic, classify finite and infinite cyclic groups and construct quotient groups, new finite groups using direct products, non-abelian groups and isomorphic groups.

| **MODULE I) INTRODUCTION TO GROUP THEORY s15 LECTURES** |
|---|
| (i) Definition and basic properties of groups. Order of a group. Order of an element. Subgroups. Criterion for a subset of a group to be a subgroup. Abelian groups. Centre of a group. Normalizer of an element in a group. Examples of groups including $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}_n$, Klein 4-group, Quaternion group, Permutation groups: symmetric groups ($S_n$) and alternating groups ($A_n$), $\mu_n$ ($n^{\text{th}}$ roots of unity), $S^1$(= the |

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|---|
| unit circle in $\mathbb{C}$), $GL_n(\mathbb{R})$, $SL_n(\mathbb{R})$, $T_n$ (= the group of $n\times n$ non-singular upper triangular matrices), $L_n$ (= the group of $n\times n$ non-singular lower triangular matrices), and groups of symmetries of plane figures.<br>(ii) Definition of group homomorphism with simple examples. Kernel and image of a group homomorphism, Fundamental theorem of group homomorphism, Second and Third isomorphism theorems of groups. |
| **MODULE II) DIRECT PRODUCTS AND CYCLIC GROUPS**<br>**15 LECTURES** |
| (i) Cosets of a subgroup in a group. Lagrange's Theorem. Normal subgroups. Listing normal subgroups of certain groups, Quotient (or Factor) groups.<br>(ii) External direct products of groups, Internal Direct Products, Examples.<br>(iii) Definition of Cyclic group, Examples. Properties of cyclic groups and cyclic subgroups. The necessary and sufficient condition for the direct product of cyclic groups to form a cyclic group.<br>(iv) Finite and infinite cyclic groups: their subgroups and generators. Properties of generators. Characterization of cyclic groups (as being isomorphic to $\mathbb{Z}$ or $\frac{\mathbb{Z}}{n\mathbb{Z}}$ for some $n \in \mathbb{N}$). Classifications of groups of order up to 7 (without proof). |

**Recommanded Books:**


1. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi, 2013.


**References Books:**


1. T. W. Hungerford. Graduate Texts in Mathematics: Algebra, Springer verlag, New York, 1974.
2. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc., U. S., 3rd edition, 2003.
3. I. S. Luther, I. B. S. Passi. Algebra. Vol. I, Narosa publication House, New Delhi, India, 1996.
4. I. S. Luther, I. B. S. Passi. Algebra. Vol. II, Narosa publication House, New Delhi, India, 1999.
5. J. A. Gallian, Contemporary Abstract Algebra, Seventh Edition, Houghton Mifflin, USA, 2009.

6. Michael Artin, Algebra, Pearson Education, India, Second Edition, 2024.
7. N. S. Gopalkrishnan, University Algebra, New Age International Private Limited, India, 2018.
8. I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition, USA,1975.
9. P. B. Bhattacharya, S. K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Cambridge University press, 1995

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTT303 **(MAJOR)** | Topology of Metric Spaces – I | 2 | Theory | 30 |

**Course Objectives:**

- To introduce students to the fundamental concepts and structures of metric spaces by studying various metrics on standard mathematical spaces and understanding the notions of open and closed sets, interior, closure, boundary, and related topological properties.
- To develop the ability to analyze sequences in metric spaces, including convergence, Cauchy sequences, completeness, and classical results such as Cantor's Intersection Theorem and its applications.
- To develop a clear understanding of the topological properties of functions and their applications.
- To build problem-solving skills through computation of distances and diameters of sets, construction of open and closed balls, identification of limit and isolated points, and formulation of examples and counterexamples illustrating key properties of metric spaces.

**Course Outcomes:** After completion of the course, students will be able to

- define and explain the basic concepts of metric spaces, including metrics, open and closed sets, interior, closure, and boundary, convergent sequences, Cauchy sequences, continuity, uniform continuity with suitable examples.
- compute the interior, closure, limit points, diameter of a given subset of a metric space, and compute the distance between two subsets and the distance of a point from a given set.
- examine convergence, Cauchy sequences, completeness, and related results such as Cantor's Intersection Theorem, and analyze their consequences in different metric spaces.
- compare and distinguish between topological structures arising from different metrics and apply Cantor's Intersection Theorem in suitable examples.
- construct open balls and closed balls in given metric space and construct counter examples for convergent/Cauchy sequences and continuous/uniformly continuous functions.

| MODULE I) METRIC SPACES AND TOPOLOGICAL STRUCTURES 15 LECTURES |
|---|
| (i) Definition and examples of metric spaces such as $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$ with its Euclidean, sup and sum metrics. $\mathbb{C}$ (Complex numbers). $l^1$ and $l^2$ spaces of sequences. C[a, b] the space of real valued continuous functions on [a, b].<br>(ii) Discrete metric space. Metric induced by the norm.<br>(iii) Metric subspaces, product of metric spaces.<br>(iv) Open balls and open sets in a metric space. Examples of open sets in various metric spaces. Properties of open sets. Hausdorff property. Interior of a set and its properties.<br>(v) Closed sets. Closed balls, Examples. Limit point of a set. Isolated point. Closure of a set and its properties. Boundary of a set. Distance of a point from a set, Distance between sets. Diameter of a set. Bounded sets. Structure of an open set in $\mathbb{R}$. Equivalent metrics. |

| MODULE II) SEQUENCES AND CONTINUITY 15 LECTURES |
|---|
| (i) Sequences in a metric space. Convergent sequence in metric space. Cauchy sequence in a metric space. Subsequences. Examples of convergent and Cauchy sequences in different metric spaces. Characterization of limit points and closure points in terms of sequences.<br>(ii) Definition of complete metric spaces. Examples of complete metric spaces. Completeness property in subspaces (Statements only)<br>(iii) Cantor's Intersection Theorem, Converse of Cantor's Intersection Theorem (Statement Only), Nested Interval theorem in $\mathbb{R}$ (without proof), Applications of Cantors Intersection Theorem (without proof) such as (a) The set of real Numbers is uncountable; (b) Density of rational Numbers; (c) Intermediate Value theorem.<br>(iv) Epsilon-delta definition of continuity. Characterization of continuity at a point in terms of sequences, open sets and closed sets and related examples. Continuity of composite function.<br>(v) Uniform continuity in a metric space, examples (emphasis on $\mathbb{R}$) Contraction mapping (fixed point) theorem. |

**Recommanded Book:**

1. S. Kumaresan; Topology of Metric spaces, Narosa Publication Hose, New Delhi, 2nd edition, 2011.

**References Books:**


1. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
2. R. R. Goldberg, Methods of Real Analysis, John Wiley and Sons, Inc., New York, 2nd Edition, 1976.
3. P. K. Jain, K. Ahmed, Metric Spaces, Narosa Publication House, New Delhi, 3rd edition, 2019.
4. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadav; An Introduction to Metric Spaces, Chapman and Hall/CRC press, New York, 1st Edition, 2020.
5. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill education, India, Third Edition, 1976.
6. D. Somasundaram; B. Choudhary; A First Course in Mathematical Analysis. Narosa publication House, New Delhi, India, 1st Edition, 1996.
7. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill company, Inc., New York, 1st Edition, 1963.
8. E. T. Copson; Metric Spaces, Cambridge University press, London, 1st edition,1968.

## EXAMINATION PATTERN FOR

### USMTT301, USMTT302, USMTT303

### Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Home Assignment / a quiz on one of the modules. | 05 Marks |
| 03 | Seminar/ group presentation on any one topic related to the syllabus. | 05 Marks |
| | **Total** | **20 Marks** |

**Paper pattern of the Test (Duration: 30 Minutes):**

Q 1: Definitions/MCQ/ Fill in the blanks/True or False with Justification. (04 Marks: 4 x 1).

Q 2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3).

### Semester End Examination (SEE): 30 Marks

| **Duration: 1 Hour** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Attempt Any Two:<br>(a) Question on Module I<br>(b) Question on Module I<br>(c) Question on Module I | 10 Marks |
| Q. 2 A) | Attempt Any Two:<br>(a) Question on Module II<br>(b) Question on Module II<br>(c) Question on Module II | 10 Marks |
| Q. 3 A) | Attempt Any Two:<br>(a) Question on Module I or Module II<br>(b) Question on Module I or Module II<br>(c) Question on Module I or Module II | 10 Marks |
| | **Total** | **30 Marks** |

**Practical - 301 (Practical based on Integral Calculus and Group Theory)**
**(USMTP304)**

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | Credits |
|---|---|---|---|---|
| **V (Level 5.5)** | **USMTP304 (MAJOR)** | **Practical - 301 (Practical based on Integral Calculus and Group Theory)** | **Practical** | **02** |

**Course Objectives (CO):**

- To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting.
- To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences.
- To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment.
- To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.

**Course Outcomes:** After the completion of the course, students will be able:

- Apply double and triple integrals for computing areas and volumes, and apply basic concepts of group theory to solve algebraic problems.
- Analyze integrability and evaluate multiple integrals using Fubini’s Theorem, and analyze fundamental group properties and homomorphisms.
- Justify/Check the validity of integration techniques and coordinate changes, and justify group-theoretic results and proofs.
- Construct appropriate coordinate systems for complex integrals, and construct groups, subgroups, and related algebraic structures.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Evaluation of Double Integration in Cartesian Coordinates |
| **2.** | Evaluation of Double Integration by reversing order of the integration |
| **3.** | Evaluation of the area using Double Integration |
| **4.** | Evaluation of Double Integration in Polar Coordinates |
| **5.** | Evaluation of Triple Integration in cartesian coordinates |
| **6.** | Evaluation of Triple Integration using cylindrical and spherical coordinates |
| **7.** | Finding Volume using Triple Integration |
| **8.** | Groups and Order of an Element in a group |
| **9.** | Centralizer of an element and center of a Group |
| **10.** | Subgroups of a group, Cosets and Lagrange's Theorem |
| **11.** | Homomorphism, Kernel, image under group homomorphism and Isomorphism of Groups |
| **12.** | Internal and External Direct Products of Groups |
| **13.** | Cyclic groups, Subgroups of Cyclic groups and their generators |
| **14.** | Product of Cyclic groups and their properties |
| **15.** | Characterization of Cyclic groups via Isomorphisms |
| | |
| | **Miscellaneous Theory questions based on both modules** |

**Recommanded Books:**


1. T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 (Section 1.1 to 11.8).
2. James Stewart, Calculus: early transcendental, Cengage learning, USA, 8th edition, 2014 - (Section 16.5 to 16.9).
3. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi, 2013.


**Reference Books:**


1. T. Apostol, Mathematical Analysis, Second Edition, Narosa, New Delhi. 1947.
2. R. Courant and F. John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.

3. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M. H. Protter and C. B. Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
5. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. D. V. Widder, Advanced Calculus, Second Ed., Dover Pub., New York, 1989.
7. T. W. Hungerford. Algebra, Springer.
8. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
9. N. S. Gopalkrishnan, University Algebra, New edge international publishers, India, New Delhi, 2nd edition, 2004.
10. Marsden and Jerrold E. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996.
11. I. N. Herstein, Topics in Algebra, John Wiley Sons, New York, Second edition, 1975.
12. P. B. Bhattacharya, S.K. Jain, S. Nagpaul, Abstract Algebra, Foundation Books, Second edition, New Delhi, 1995.

**Practical - 302**

**(Practical based on Integral Calculus and Topology of Metric Spaces - I)**

**(USMTP305)**

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | Credits |
|---|---|---|---|---|
| V (Level 5.5) | USMTP305 (MAJOR) | Practical - 302 (Practical based on Integral Calculus and Topology of Metric Spaces) | Practical | 02 |

**Course Objectives:**

- To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting.
- To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences.
- To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment.
- To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Apply concepts of scalar and vector fields, vector differential operators, and multiple integrals to compute circulation, flux, and work in $\mathbb{R}^2$ and $\mathbb{R}^3$, and apply basic concepts of metric spaces such as open and closed sets, bounded sets, and closure.
- Analyse parametrized curves and surfaces to formulate and interpret line and surface integrals, and analyse limit points and convergence in metric spaces.

- Justify / Check whether a given vector field is conservative and verify Green's, Stokes', and Gauss' theorems, and justify completeness and continuity in metric spaces.
- Construct suitable parametrizations of curves and surfaces to solve problems involving circulation and flux, and distinguish between continuity and uniform continuity in metric spaces.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Line Integrals of Scalar Fields |
| **2.** | Line Integrals of Vector Fields |
| **3.** | Evaluation of Work done using line integral |
| **4.** | Green's Theorem |
| **5.** | Surface Integrals |
| **6.** | Stoke's Theorem |
| **7.** | Gauss Divergence Theorem |
| **8.** | Metric Spaces, Subspaces and Normed Linear Spaces |
| **9.** | Sketching of Open Balls in $\mathbb{R}2$, Open and Closed sets |
| **10.** | Interior of a set, Interior points and Limit Points. |
| **11.** | Closure of a set, Dense Sets, Separability and Diameter of a set |
| **12.** | Bounded, Convergent and Cauchy Sequences |
| **13.** | Complete Metric Spaces |
| **14.** | Cantor's Intersection Theorem and Applications |
| **15.** | Continuity and Uniform Continuity |
| | |
| | **Miscellaneous Theory questions based on both modules** |

**Recommanded Books:**

1. T. M. Apostol, Calculus, Vol. 2, Second Edition., John Wiley, New York, 1969 (Section 1.1 to 11.8).
2. James Stewart, Calculus Early Transcendentals, 8th Edition, Cengage Learning, USA, 2014. (Section 16.5 to 16.9).
3. S. Kumaresan; Topology of Metric spaces, Narosa Publication Hose, New Delhi, 2nd edition, 2011.

**References Books:**

1. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
2. R. R. Goldberg, Methods of Real Analysis, John Wiley and Sons, Inc., New York, 2nd Edition, 1976.
3. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadav; An Introduction to Metric Spaces, Chapman and Hall/CRC press, New York, 1st Edition, 2020.
4. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill education, India, Third Edition, 1976.
5. D. Somasundaram; B. Choudhary; A First Course in Mathematical Analysis. Narosa publication House, New Delhi, India, 1st Edition, 1996.
6. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill company, Inc., New York, 1st Edition, 1963.
7. E. T. Copson; Metric Spaces, Cambridge University press, London, 1st edition,1968.
8. Ravi P. Agarwal and Donal O’Regan; Ordinary and Partial Differential Equations: With Special Functions, Fourier Series, and Boundary Value Problems, Springer publisher, New York, First Edition, 2009.
9. W. E. Williams; Partial Differential Equations; Clarendon Press, Oxford, First Edition, 1980.
10. K. Sankara Rao, Introduction to Partial Differential Equations; PHI learning Pvt. Ltd., Third Edition, 2011.
11. P. K. Jain, K. Ahmed; Metric Spaces; Narosa, New Delhi, 1996.

## EXAMINATION PATTERN FOR
## USMTP304, USMTP305

### Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Overall performance | 05 Marks |
| 03 | Viva | 05 Marks |
| | **Total** | **20 Marks** |

### Semester End Examination (SEE): 30 Marks

| **Duration: 2 Hours** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Multiple choice questions (Attempt any 5 from 8). | 10 Marks |
| Q. 2 A) | Attempt any Three out of Five. | 15 Marks |
| | **Total** | **25 Marks** |

**Journal: 5 Marks**

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course Title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTET306A **(ELECTIVE)** | Graph Theory – I | 2 | Theory | 30 |

**Course Objectives:**

- To provide a strong foundation in the basic concepts, definitions, and properties of graphs, enabling students to model and interpret relationships in diverse systems.
- To develop proficiency in essential graph-theoretic tools, preparing students for further study in mathematics, computer science, optimization, and network analysis.
- To enhance analytical and problem-solving abilities by engaging with graph algorithms, logical structures, and practical applications relevant to modern technological domains.
- To encourage awareness of the role of graph theory in addressing global and local challenges such as communication networks, logistics, data processing, and cybersecurity.

**Course Outcomes:** After completion of the course, students will be able to

- Understand and recall fundamental concepts such as graph types, connectivity, degree sequences, graph isomorphism, matrices associated with graphs, and properties of trees and Eulerian graphs.
- Apply graph-theoretic techniques and algorithms—including BFS, DFS, Dijkstra's, Kruskal's, and Prim's—to solve problems related to shortest paths, spanning trees, network design, and optimization.
- Analyze the structural properties of graphs, examine conditions for bipartiteness, evaluate graph centers and eccentricities, and interpret the behavior of connectivity, cut edges, and cut vertices.
- Justify structural characteristics using the Havel–Hakimi theorem, Eulerian criteria, and tree characterizations, and verify correctness of graph-based algorithms.
- Construct examples and counterexamples related to degree sequences, bipartite graphs, spanning trees, Eulerian and non-Eulerian graphs and minimal spanning trees

| MODULE I) BASICS OF GRAPHS 15 LECTURES |
|---|
| (i) Definition of general graph, Directed and undirected graph, Simple and multiple graphs, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs, Subgraph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs.<br>(ii) Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self-complementary graphs, connected graphs, Connected components.<br>(iii) Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths.<br>(iv) Degree sequence and Havel-Hakimi theorem (statement only), Center and eccentricity of a graph. Distance in a graph- shortest path problems, Dijkstra's algorithm (statement only). |
| **MODULE II) TREES, EULERIAN GRAPHS 15 LECTURES** |
| (i) Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations.<br>(ii) Spanning tree and co-tree, Recurrence relation of spanning trees and Cayley formula (statement only) for spanning trees of $K_n$, Fundamental Circuits and Fundamental Cut Sets.<br>(iii) Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs and minimal spanning trees – Kruskal's and Prim's algorithms (without proof) for minimal spanning trees.<br>(iv) Eulerian graph and its characterization, Fleury's Algorithm (without proof). |

**Recommanded Books:**


1. J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Elsevier Science Publishing Co., Inc., New York, 5th Edition, 1982.
2. R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer-Verlag, New York, 2012.


**Reference Books:**


1. Douglas B. West, Introduction to Graph Theory, 2nd Edition, Pearson College Div publisher, 2000.
2. Santanu Saha Ray, Graph Theory with Algorithms and its Applications

in Applied Science and Technology, Springer 2013.
3. Mehadi Behzad and Gary Chartrand, Introduction to Theory of Graphs. Allyn & Bacon publisher, 1971.
4. S. A. Choudam, A First Course in Graph Theory, Laxmi Publication, 2000.

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTET306B **(ELECTIVE)** | Number Theory and Its applications – I | 2 | Theory | 30 |

**Course Objectives:**

- To develop a solid understanding of core principles and methods in number theory, enabling students to analyze and solve problems involving integers and congruences.
- To illustrate the depth and scope of number theory by equipping students with essential theoretical tools required for further studies in pure and applied mathematics.
- To strengthen problem-solving abilities and logical reasoning through rigorous proofs and structured arguments, fostering clarity of mathematical expression.
- To provide exposure to classical results and foundational ideas in number theory that underpin significant developments within the broader Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Understand and recall fundamental concepts of number theory, including divisibility, primes, congruences, arithmetic functions, and basic factorization methods.
- Apply standard theorems and techniques of number theory to solve problems involving linear and higher-degree congruences and classical Diophantine equations.
- Analyze and interpret integer solutions of equations, residue systems, and congruence relations using appropriate theoretical tools and logical reasoning.
- Justify results related to solvability or non-solvability of Diophantine equations and congruences through rigorous mathematical proofs.
- Construct and examine examples and counterexamples illustrating key concepts and theorems in number theory, thereby demonstrating deeper conceptual understanding.

| **MODULE I) CONGRUENCES AND FACTORIZATION 15 LECTURES** |
|---|
| (i) Review of Divisibility, Primes and The fundamental theorem of Arithmetic.<br>Congruences: Definition and elementary properties, Complete residue system |

| |
|---|
| modulo m, Reduced residue system modulo m.<br>(ii) Euler's function and its properties, Fermat's little Theorem, Eulers generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of Higher degree. The Fermat-Kraitchik Factorization Method. |
| **MODULE II) DIOPHANTINE EQUATIONS AND THEIR SOLUTIONS 15 LECTURES** |
| (i) The linear equations ax + by = c. The equations $x^2 + y^2 = p$, where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions.<br>(ii)The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x; y; z) with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples. |

**Recommended Books:**

1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc., 1991.
2. David M. Burton, Elementary Number Theory, Tata McGraw-Hill Edition, 7th Edition, 2023.

**References Books:**

1. N. S. Gopalkrishnan: University Algebra, New Age International Ltd, Reprint 2013.
2. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
3. Neville Robins, Beginning Number Theory; Narosa Publications.
4. S. D. Adhikari; An introduction to Commutative Algebra and Number Theory; Narosa Publishing House.
5. N. Koblitz; A course in Number theory and Cryptography; Springer. 7. M. Artin; Algebra; Prentice Hall.
6. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory; Second edition, Springer Verlag.
7. William Stalling; Cryptology and network security.
8. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.

**EXAMINATION PATTERN FOR**
**USMTET306A, USMTET306B**

## Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Home Assignment / a quiz on one of the modules. | 05 Marks |
| 03 | Seminar/ group presentation on any one topic related to the syllabus. | 05 Marks |
| | **Total** | **20 Marks** |

**Paper pattern of the Test (Duration: 30 Minutes):**

Q 1: Definitions/MCQ/ Fill in the blanks/True or False with Justification. (04 Marks: 4 x 1).

Q 2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3).

## Semester End Examination (SEE): 30 Marks

| **Duration: 1 Hour** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Attempt Any Two:<br>(b) Question on Module I<br>(b) Question on Module I<br>(c) Question on Module I | 10 Marks |
| Q. 2 A) | Attempt Any Two:<br>(a) Question on Module II<br>(b) Question on Module II<br>(c) Question on Module II | 10 Marks |
| Q. 3 A) | Attempt Any Two:<br>(d) Question on Module I or Module II<br>(e) Question on Module I or Module II<br>(f) Question on Module I or Module II | 10 Marks |
| | **Total** | **30 Marks** |

**Practical - 303 (Practical based on Graph Theory - I)**

**(USMTEP307A)**

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| **V (Level 5.5)** | **USMTEP307A (ELECTIVE)** | **Practical - 303 (Practical based on Graph Theory - I)** | **Practical** | **02** |

**Course Objectives (CO):**

- To develop the ability to construct, classify, and analyze various types of graphs and their substructures using foundational graph-theoretic definitions and properties.
- To enable students to compute and interpret key graph parameters-degrees, connectivity, distances, centers, eccentricities, and degree sequences-while applying the Handshaking Theorem, Havel Hakimi theorem, and matrix representations of graphs.
- To build proficiency in identifying graph isomorphism, verifying structural similarities, and applying shortest-path and traversal algorithms such as Dijkstra's, BFS, and DFS.
- To cultivate practical skills in working with trees, spanning trees, cut vertices, cut edges, fundamental circuits, and applying classical results including Cayley's formula and minimal spanning tree algorithms (Kruskal's and Prim's).
- To strengthen students' problem-solving abilities by applying Eulerian graph characterizations, Fleury's algorithm, Huffman coding, and tree-based structures in modelling, optimizing, and interpreting real-world network scenarios.

**Course Outcomes (OC):** After the completion of the course, students will be able:

- Construct, classify, and analyze various types of graphs and subgraphs using fundamental graph-theoretic definitions, terminology, and structural properties.
- Apply graph algorithms and matrix representations to compute degrees, identify connectivity, examine isomorphism, and verify degree sequences using methods such as the Havel–Hakimi theorem.

- Implement and evaluate shortest-path and spanning-tree algorithms-including Dijkstra's, BFS, DFS, Kruskal's, and Prim's-to solve optimization and network-based problems.
- Analyze and interpret the properties of trees, cut vertices, cut edges, Eulerian graphs, and Huffman coding to develop efficient solutions for routing, encoding, and network design tasks.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Classification of Graphs, subgraphs, Degree Analysis and Handshaking Theorem. |
| **2.** | Walks, Trails, Paths, Circuits, Cycles and Self-complementary graphs. |
| **3.** | Connected graphs, Center, eccentricity and distance in a graph. |
| **4.** | Isomorphism of Graphs, Matrices associated with the graphs. |
| **5.** | Self-complementary graphs, Connected graphs. |
| **6.** | Degree sequence and Havel-Hakimi theorem. |
| **7.** | Shortest path problems, Dijkstra's algorithm. |
| **8.** | Definition of a tree and its characterizations. |
| **9.** | Cut vertices. Cut edges and its Characterization. |
| **10.** | Spanning tree and co-tree, Recurrence relation of spanning trees. |
| **11.** | Cayley formula for spanning trees, Fundamental Circuits and Cut Sets. |
| **12.** | Algorithms for spanning tree-BFS and DFS. |
| **13.** | Binary and m-ary tree, Prefix codes and Huffman coding. |
| **14.** | Kruskal's and Prim's algorithm for minimal spanning trees. |
| **15.** | Eulerian graph and its characterization, Fleury's Algorithm. |

**Recommanded Books:**


1. J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Elsevier Science Publishing Co., Inc., New York, 5th Edition, 1982.
2. R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer-Verlag, New York, 2012.


**Reference Books:**


1. Douglas B. West, Introduction to Graph Theory, 2nd Edition, Pearson College Div publisher, 2000.
2. Santanu Saha Ray, Graph Theory with Algorithms and its Applications

in Applied Science and Technology, Springer 2013.

3. Mehadi Behzad and Gary Chartrand, Introduction to Theory of Graphs. Allyn & Bacon publisher, 1971.
4. S. A. Choudam, A First Course in Graph Theory, Laxmi Publication, 2000.

**Practical - 304 (Practical based on Number Theory and Its applications - I)**

**(USMTEP307B)**

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| V (Level 5.5) | USMTEP307B (ELECTIVE) | Practical - 304 (Practical based on Number Theory and Its applications - I) | Practical | 02 |

**Course Objectives:**

- To develop a solid understanding of core principles and methods in number theory, enabling students to analyze and solve problems involving integers and congruences.
- To illustrate the depth and scope of number theory by equipping students with essential theoretical tools required for further studies in pure and applied mathematics.
- To strengthen problem-solving abilities and logical reasoning through rigorous proofs and structured arguments, fostering clarity of mathematical expression.
- To provide exposure to classical results and foundational ideas in number theory that underpin significant developments within the broader Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Apply standard theorems and techniques of number theory to solve problems involving linear and higher-degree congruences and classical Diophantine equations.
- Analyze and interpret integer solutions of equations, residue systems, and congruence relations using appropriate theoretical tools and logical reasoning.
- Justify results related to solvability or non-solvability of Diophantine equations and congruences through rigorous mathematical proofs.
- Construct and examine examples and counterexamples illustrating key concepts and theorems in number theory, thereby demonstrating deeper conceptual understanding.

# LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Divisibility and congruences. |
| **2.** | Residue systems modulo m. |
| **3.** | Euler's function, Fermat's little Theorem |
| **4.** | Euler's generalization of Fermat's little Theorem, Wilson's theorem. |
| **5.** | Linear congruence. |
| **6.** | The Chinese remainder Theorem. |
| **7.** | Congruences of Higher degree. |
| **8.** | The Fermat-Kraitchik Factorization Method. |
| **9.** | The Linear equations $ax + by = c$. |
| **10.** | The equations $x^2 + y^2 = p$, where p is a prime. |
| **11.** | The diophantine equation $x^2 + y^2 = z^2$. |
| **12.** | Problems on Pythagorean triples, primitive solutions. |
| **13.** | Problems on the equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ |
| **14.** | Positive integer n can be expressed as a sum of squares of four integers. |
| **15.** | Assorted examples. |
| | |
| | **Miscellaneous Theory questions based on both modules.** |

**Recommended Books:**


1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc., 1991.
2. David M. Burton, Elementary Number Theory, Tata McGraw-Hill Edition, 7th Edition, 2023.


**References Books:**


1. N. S. Gopalkrishnan: University Algebra, New Age International Ltd, Reprint 2013.
2. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
3. Neville Robins, Beginning Number Theory; Narosa Publications.
4. S. D. Adhikari; An introduction to Commutative Algebra and

Number Theory; Narosa Publishing House.
5. N. Koblitz; A course in Number theory and Cryptography; Springer. 7. M. Artin; Algebra; Prentice Hall.
6. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory; Second edition, Springer Verlag.
7. William Stalling; Cryptology and network security.
8. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.

**EXAMINATION PATTERN FOR**

**USMTEP307A, USMTEP307B**

**Continuous Internal Evaluation (CIE): 20 Marks**

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Overall performance | 05 Marks |
| 03 | Viva | 05 Marks |
| | **Total** | **20 Marks** |

**Semester End Examination (SEE): 30 Marks**

| **Duration: 2 Hours** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Multiple choice questions (Attempt any 5 from 8). | 10 Marks |
| Q. 2 A) | Attempt any Three out of Five. | 15 Marks |
| | **Total** | **25 Marks** |

**Journal: 5 Marks**

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | MTVS04 **(VSC)** | Computational Machine Learning | 2 | Practical | 60 |

**Course Objectives:**

- To understand the machine learning, its uses and how it changes day-to-day life.
- To apply Python programming language tool like Panda to write a simple program.
- To visualizing the important characteristics of data using graphs and plots drawn in Matplotlib.
- Analysis of data using tools of descriptive statistics and probability distribution.
- Interpretation of data using various tests of Hypothesis and techniques of ANOVA.
- Building Machine learning modules on different datasets using the Gradient Descent algorithm, Scikit-Learn.
- To develope various means to ascertain the validation of the module created with the above techniques.

**Course Outcomes:** After completion of the course, students will be able to

- Understand machine learning concepts and how it influences our day-to-day activities.
- Highlights key features of data using various techniques of descriptive statistics and by plotting the graphs and plots.
- Build a predictive module on various datasets using different tools of Machine learning.
- Validate the module created by them for accuracy and better prediction.

# LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Demonstration Machine learning, different types of Machine learning algorithm with real life example. |
| **2.** | Python Revisit: Basic of python, conditional and loop statement, list, tuples and dictionary. |
| **3.** | Revisiting Panda: Introduction of Jupyter Notebook, Creation of panda dataframe, loading the file of different format, rows and columns operations. |
| **4.** | Matplotlib for Creation of graphs like bar graph, histogram, Box plot, scatter plot, pair plot, heatmap. |
| **5.** | Descriptive statistics with Python, Computation of Measures of central tendency and dispersions. |
| **6.** | Computation of standard probability distribution using Python, Chi-squared test and analysis of variance with Python. |
| **7.** | Building of simple regression model. |
| **8.** | Validation of simple regression model. |
| **9.** | Demonstration of classification problems with real life examples. |
| **10.** | Creation of Binary logistic regression for German credit rating dataset. |
| **11.** | Validation of Binary logistic regression created for German credit rating dataset. |
| **12.** | Creation of Linear regression model using Gradient Descent (GD) algorithm. |
| **13.** | Validation of Linear regression model created using GD algorithm. Programs to illustrate the concept of method overriding and use of super keyword. |
| **14.** | Demonstration of Scikit-Learn library and its uses. Explanation of Iris dataset. |
| **15.** | Supervised learning examples: Simple linear regression, Iris classification. Unsupervised learning examples: Iris dimensionality and Iris clustering. |

**Recommended Books:**


1. (Practical 1 to 13): Manaranjan Pradhan, U Dinesh Kumar, Machine learning using Python, Wiley publishers, India, 2019.
2. (Practical 14 to 15): Jake VanderPlas, Python Data Science Handbook, Essential Tools for Working with Data, O'Reilly publisher, First Edition, 2016.

**References Books:**

1. Wes Mckinney, Python for Data Analysis: Data Wrangling with Pandas, NumPy and IPython, O'Reilly publisher, Second edition, 2017.
2. Bernd Klein, Data Analysis Numpy, Matplotlib and Pandas, 2021.
3. Andreas C. Müller and Sarah Guido, Introduction to Machine Learning with Python A Guide for Data Scientists, O'Reilly publisher, First Edition, 2017.

## EXAMINATION PATTERN

### Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Overall performance | 05 Marks |
| 03 | Viva | 05 Marks |
| | **Total** | **20 Marks** |

### Semester End Examination (SEE): 30 Marks

| **Duration: 2 Hours** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Multiple choice questions (Attempt any 5 from 8). | 10 Marks |
| Q. 2 A) | Attempt any Three out of Five. | 15 Marks |
| | **Total** | **25 Marks** |

**Journal: 5 Marks**

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| V | USMTT309 **(MINOR)** | Ordinary Differential equations | 2 | Theory | 30 |

## Course Objectives:

- To give sufficient knowledge of First order first degree differential equations.
- To learn various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.
- Apply theoretical knowledge to practical situations.
- To understand the Homogeneous and non-homogeneous second order linear differentiable equations.

## Course Outcomes: After completion of the course, students will be able to

- Understand the genesis of First order first degree differential equations.
- Learn various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.
- Apply the knowledge to solve the real-life problems.
- Understand the Homogeneous and non-homogeneous second order linear differentiable equations.

| **MODULE I) FIRST ORDER FIRST DEGREE DIFFERENTIAL EQUATIONS 15 LECTURES** |
|---|
| (i) Definitions of Differential Equation, Order and Degree of a Differential Equation, ordinary differential equation (ODE), Partial Differential Equation, Linear ODE, Bernoulli's differential equation. Definition of Lipschitz function, examples. Existence and Uniqueness Theorem for the differential equation $y' = f(x, y)$, $y(x_0) = y_0$ where $f(x, y)$ is a continuous function satisfying Lipschitz condition $\|f(x, y_1) - f(x, y_2)\| \leq K\|y_1 - y_2\|$ on the strip $a \leq x \leq b$ & $y \in R$ (statement |

| |
|---|
| only). (ii) Exact Equations: General Solution of Exact equations of first order and first degree, Necessary and sufficient condition for M dx + N dy = 0 to be exact. Linear and reducible to linear equations, Applications of first order ODE viz. orthogonal trajectories, population growth, and finding the current at a given time. |
| **MODULE II) SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS 15 LECTURES** |
| (i) Existence and uniqueness theorems to be stated clearly when needed in the sequel. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. (ii) The general solution of homogeneous differential equation. The use of known solutions to find the general solution of homogeneous equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals. The homogeneous equation which constant coefficient, auxiliary equation. (iii) The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters. |

**Recommended Books:**


1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill publisher, 2nd Edition, 2017.
2. E. A. Coddington, An introduction to ordinary differential equations, Dover Publications Inc., 1989.


## Reference Books:


1. Ravi P. Agarwal and Donal O'Regan; Ordinary and Partial Differential Equations: With Special Functions, Fourier Series, and Boundary Value Problems, Springer publisher, New York, First Edition, 2009.
2. K. Sankara Rao, Introduction to Partial Differential Equations; PHI learning Pvt. Ltd., Third Edition, 2011.
3. Ian Sneddon, Elements of Partial Differential Equations, Dover publication Inc., New York, 1st edition, 2006.

## EXAMINATION PATTERN

### Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Home Assignment / a quiz on one of the modules. | 05 Marks |
| 03 | Seminar/ group presentation on any one topic related to the syllabus. | 05 Marks |
| | **Total** | **20 Marks** |

### Paper pattern of the Test (Duration: 30 Minutes):

Q 1: Definitions/MCQ/ Fill in the blanks/True or False with Justification. (04 Marks: 4 x 1).

Q 2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3).

### Semester End Examination (SEE): 30 Marks

| **Duration: 1 Hour** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Attempt Any Two:<br>(c) Question on Module I<br>(b) Question on Module I<br>(c) Question on Module I | 10 Marks |
| Q. 2 A) | Attempt Any Two:<br>(a) Question on Module II<br>(b) Question on Module II<br>(c) Question on Module II | 10 Marks |
| Q. 3 A) | Attempt Any Two:<br>(a) Question on Module I or Module II<br>(b) Question on Module I or Module II<br>(c) Question on Module I or Module II | 10 Marks |
| | **Total** | **30 Marks** |

## STRUCTURE OF THE COURSE

| Semester | Cours Code | Course title | No. of Credits | Duration |
|---|---|---|---|---|
| V | MTCE P-01 | Community engagement programme | 2 | 30 hrs (Field Work + Survey) + 15hrs (Discussion + Report Writing) **: Total - 45 hrs** |

## Indicative Topics for CEP

| Sr. No. | Name of the Topic |
|---|---|
| 1 | Mathematics of Insurance and Risk Assessment for Community Awareness |
| 2 | Statistical Study of Employment and Unemployment Trends in the Local Area |
| 3 | Mathematical Modelling of Queueing Systems in Banks and Public Offices |
| 4 | Survey and Statistical Analysis of Digital Literacy in the Community |
| 5 | Mathematics Behind Pricing Strategies and Profit Optimization for Small Vendors |
| 6 | Application for Linear Programming in Resource Allocation for Community Welfare Programs |
| 7 | Mathematical Study of Rainfall Patterns and Their Impact on Agriculture |
| 8 | Mathematical Modelling of Population Growth and Migration Trends in the Town. |
| 9 | Using Mathematics to Study Traffic Flow and Road Safety in Urban Areas. |
| 10 | Mathematics Olympiad Training and Mentorship Program for School Student. |
| 11 | Community Mathematics Festival: Promoting Mathematical Awareness and Engagement. |
| 12 | Integration of Mathematics and Art: A Community Exhibition and Study |

| | |
|---|---|
| 13 | Mathematics-Based Game Development as a Tool for Learning and Engagement. |
| 14 | Statistical and Mathematical Analysis of Cryptocurrency Market Trends |
| 15 | Activity-Oriented Mathematics Education for Enhancing Numerical and Financial Literacy among Underprivileged Students. |
| 16 | Mathematics-Based Academic Support Programs for Children from Economically Weaker Sections. |
| 17 | Demographic Data Analysis of the Local Community Using Mathematical Tools. |
| 18 | A Mathematical Investigation of Sleep Patterns and Their Impact on Academic Performance. |
| 19 | Applying Statistics to Analyze Nutrition and Health Trends in the Local Community. |
| 20 | Survey-Based Statistical Analysis of Social and Community Issues |
| 21 | Mathematical Study of Household Waste Generation and Reduction Strategies. |
| 22 | Mathematical Modelling for Effective Water Resource Management in the Town. |
| 23 | Optimization of Public Transportation Systems Using Mathematical Techniques |
| 24 | Mathematical Modelling of Economic Growth Patterns of the Town |
| 25 | Statistical Analysis of Local Election Results and Voting Trends |

The topics are indicative and the faculty members should allot Community Engagement Project that are relevant and important as per core Subject. The Community Engagement Project may be taken dividual or in a group up to 5 students with proper guidance from Faculty.

## Evaluation Pattern:

Evaluation during CEP Program involves two key components: -

External Evaluation 60%

Internal Evaluation 40%

# Evaluation Chart

**(i) External Evaluation (Marks 30)**

| **Criteria** | **Marks** |
|---|---|
| Objectives, Literature Review, Methodology, Data Analysis, Conclusion and Recommendations | 15 |
| Overall Project Report Structure and Style | 05 |
| Presentation Skills and communication | 10 |
| Total | 30 |

**(ii) Internal Evaluation by Guide (Marks 20)**

| **Criteria** | **Marks** |
|---|---|
| Attendance, Community interactions completion and interaction with Supervisor. | 10 |
| Overall Report quality | 10 |
| Total | 20 |

# SEMESTER - VI

**STRUCTURE OF THE COURSE**

| Semester | Course code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| VI | USMTT309 **(MAJOR)** | Introduction to Complex Analysis | 2 | Theory | 30 |

## Course Objectives:

- To provide a solid understanding of fundamental principles and methods, equipping students with the skills to apply mathematical ideas and tools through modeling, solving, and interpretation.
- To illustrate the expansive nature of the subject by fostering the acquisition of essential mathematical tools for continued studies across various scientific fields.
- To foster students' comprehensive development by placing emphasis on problem-solving skills, nurturing creative talents, and enhancing communication abilities, all of which are vital for a range of employment opportunities.
- To ensure exposure to both global and local issues within the realm of Mathematical Sciences, allowing learners to explore diverse aspects of the discipline.

## Course Outcomes: After completion of the course, students will be able to

- Understand and remember the fundamental concepts of complex analysis, including complex numbers, the complex plane, polar coordinates, and basic properties of analytic functions, such as Cauchy-Riemann equations and Mobius transformations.
- Apply the principles of complex integration, including evaluating line integrals along various paths, using Cauchy's theorem and Cauchy's integral formula, and expanding functions into Taylor and Laurent series with appropriate radius and disc of convergence.
- Analyze the properties and behavior of analytic functions, including their harmonic properties, singularities, and residues, and investigate their application to complex power series and singularity classification (removable, pole, or essential singularities).
- Justify/ check the validity of statements regarding analytic functions, continuity, and differentiability in the complex domain using the Cauchy-Riemann equations.

- Construct counter examples to demonstrate the failure of various properties in complex analysis, such as showing the non-analytic nature of a function at a given point, or constructing examples where power series do not converge within a certain radius, or Laurent series expansion fails to exist at an isolated singularity.

| **MODULE I) INTRODUCTION TO COMPLEX ANALYSIS** **15 LECTURES** |
|---|
| **(i)** Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, $\mathbb{C}$ as a metric space (No questions to be asked), bounded and unbounded sets, point at infinity-extended complex plane, sketching of set-in complex plane.<br>(ii) Convergence of sequences of complex numbers and related results. Limit of a function f : $\mathbb{C} \rightarrow \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions.<br>(iii) Derivative of f : $\mathbb{C} \rightarrow \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, Algebra of analytic function (without proof), chain rule, basic results of analytic functions like, if f(z) = 0 everywhere in a domain D, then $f$(z) must be constant throughout D.<br>(iv) Harmonic functions and harmonic conjugate.<br>(v) Evaluation the line integral $\int f(z)dz$ over a line segment, parabolic and circular path, Cauchy' theorem. |
| **MODULE II) COMPLEX INTEGRATION, COMPLEX POWER SERIES AND LAURENT'S SERIES** **15 LECTURES** |
| (i) Mobius transformations: definition and examples. Exponential function, its properties. trigonometric functions and hyperbolic functions.<br>(ii) Taylor's theorem for analytic function and examples.<br>(iii) Power series of complex numbers and related results. Radius of convergences, disc of convergence, uniqueness of series representation, examples.<br>(iv) Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, examples. |

**Recommanded Book:**


1. J.W. Brown and R.V. Churchill, Complex analysis and Applications: Sections 18, 19, 20, 21, 23, 24, 25, 28, 33, 34, 47, 48, 53, 54, 55, Chapter 5, page 231 section 65, define residue of a function at a pole using Theorem in section 66 page 234, Statement of Cauchy's residue theorem on page 225, section 71 and 72 from chapter 7.


**Reference Books:**


1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable, Orient Blackswan Private Limited - New Delhi, 3rd Edition, 2011.
2. T.W. Gamelin, Complex analysis, Springer Verlag publisher, New York, 2001.
3. R. P. Agarwal, Kanishka Perera, Sandra Pinelas, An introduction to Complex analysis, Springer Verlag publisher, New York, 2011.
4. Mohammad Ibrahim Mir, Complex Analysis: An Introduction, Atlantic Publishers & Distributors (P) Ltd, 2025.

**STRUCTURE OF THE COURSE**

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs.) |
|---|---|---|---|---|---|
| VI | USMTT310 | Ring Theory | 2 | Theory | 30 |

## Course Objectives:

- To understand fundamental concepts of rings, including integral domains and fields.
- To explore ideals, quotient rings, prime ideals, maximal ideals and their interrelations in the context of commutative rings.
- To develop proficiency in the ring homomorphisms, isomorphisms, and factorization in rings.
- To apply abstract concepts to concrete examples, such as polynomial rings and specific integral domains, and various factorization properties in determining the structure of rings.

**Course Outcomes:** After completion of the course, students will be able to

- explain rings, integral domains, fields, ideals, quotient rings, homomorphisms, isomorphisms, and their basic properties.
- identify subrings, units, ideals, prime ideals, maximal ideals and compute the characteristic of a ring.
- form sums/products of ideals, and characterize and identify prime and maximal ideals via their quotient rings.
- compute kernels and images of homomorphism, apply the isomorphism theorems and contrast between Euclidean domains, principal ideal domains, and unique factorization domains with examples.
- apply division algorithm, analyze divisibility, irreducible, prime elements, use of Eisenstein's criterion and construct quotient rings and counter-examples of Euclidean domains, principal ideal domains, and unique factorization domains.

| **MODULE I) RINGS AND IDEALS** | **15 LECTURES** |
|---|---|
| (i) Definition and elementary properties of rings (where the definition should include the existence of unity), commutative rings, integral domains and fields. Examples, including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}/n\mathbb{Z}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{-5}\,]$, $\mathbb{Z}[X], \mathbb{R}[X], \mathbb{C}[X], \mathbb{Z}/n\mathbb{Z}[X]$. Subring of a ring.<br>(ii) Description of the units in various rings. Results such as: Field is an integral domain, A finite integral domain is a field. $\mathbb{Z}/p\mathbb{Z}$, where p is a prime, as an example of a finite field. Characteristic of a ring. Examples. Elementary facts such as: the characteristic of an integral domain is either 0 or a prime number. Units in a ring. The multiplicative group of units in a ring R [ and, in particular, the multiplicative group F∗ of non-zero elements of a field F].<br>(iii) Ideals in a ring. Sums and products of ideals. Quotient rings. Examples. Prime ideals and maximal ideals. Prime and maximal ideals in various rings. | |
| **MODULE II) ISOMORPHISM AND FACTORIZATION** | **15 LECTURES** |
| (i) Homomorphisms and isomorphism of rings. Kernel and the image of a homomorphism. Fundamental Theorem of homomorphism of a ring. Second and third isomorphism Theorem of rings (without proof). Notions of Euclidean domain (ED), principal ideal domain (PID). Examples such as $\mathbb{Z}$, $\mathbb{Z}[i]$ and polynomial rings. Relation between these two notions (ED ⇒ PID).<br>(ii) Divisibility in a ring. Irreducible and prime elements. Examples. Division algorithm in F[X] (where F is a field). Monic polynomials, greatest common divisor of f(x), g(x) ∈ F[X] (not both 0) (Examples only). Relatively prime polynomials in F[X], irreducible polynomial in F[X]. Examples of irreducible polynomials in $(\mathbb{Z}/p\mathbb{Z})$[X] (p prime), Eisenstein Criterion (without proof).<br>(iii) Notion of unique factorization domain (UFD). Elementary properties. Results (without proof) :<br>(a) $\mathbb{Z}[\sqrt{-5}\,]$ is not a UFD.<br>(b) Every PID is UFD.<br>(c) $\mathbb{Z}\,[X]$ is UFD but not a PID.<br>**(d)** If R is a UFD then R[X] is UFD. | |

**Recommanded Book:**

1. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi, 2013.

**References Books:**


1. Michael Artin, Algebra, Pearson Education, India, Second Edition, 2024.
2. J. A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa Publishing House, 2008.
3. S. Adhikari; An Introduction to Commutative Algebra and Number theory, Narosa publication House, New Delhi, India, First Edition, 2001.
4. T. W. Hungerford, Graduate Texts in Mathematics: Algebra, Springer verlag, New York, 4th Edition 2003.
5. I. S. Luther, I. B. S. Passi. Algebra. Vol. II, Narosa publication House, New Delhi, India, 1999.
6. N. S. Gopalkrishnan, University Algebra, New Age International Private Limited, India, 2018.
7. Israel Herstein; Topics in Algebra; Second edition, Wiley Eastern Limited, Second edition, 2006.
8. P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul. Abstract Algebra, Second edition, Cambridge University press, 1995.

## STRUCTURE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| VI | USMTT311 **(MAJOR)** | Topology of Metric Spaces – II | 2 | Theory | 30 |

## Course Objectives:

- To introduce students to the core concepts of compactness in metric spaces by studying open covers, equivalent characterizations, and classical results such as the Heine–Borel and Bolzano–Weierstrass properties.
- To develop the ability to analyze and prove fundamental properties of compact sets, including closedness, boundedness, and behavior under continuous mappings, with emphasis on theory and applications.
- To develop a clear and rigorous understanding of connected and path-connected spaces through separation concepts, characterization theorems, and the relationship between connectedness and continuity.
- To build problem-solving skills through the identification of compact, connected, and path-connected sets; construction of illustrative examples and counter examples.

**Course Outcomes:** After completion of the course, students will be able to

- recall and explain compact sets, connected sets, path-connected sets, and identify their examples in various metric spaces.
- apply properties of compact sets and their equivalent characterizations, such as Heine Borel and Bolzano–Weierstrass, in real metric spaces.
- examine connectedness and path connectedness in certain metric spaces through its properties and characterizations.
- analyze path-connectedness and evaluate its relationship with connectedness in different metric spaces.
- determine connected components of metric spaces and construct examples illustrating differences between connectedness, path-connectedness, and counter examples to compactness.

| MODULE I) COMPACTNESS 15 LECTURES |
|---|
| (i) Definition of a compact metric space using open cover. Examples of compact sets in different metric spaces such as $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$ with Euclidean metric.<br>(ii) Properties of compact sets: A compact set is closed and bounded, (Converse is not true). A closed subset of a compact set is compact. Union and Intersection of Compact sets.<br>(iii) Equivalent characterizations of compactness in $\mathbb{R}$ with usual metric:<br>(a) Sequentially compactness property.<br>(b) Heine-Borel property.<br>(c) Closed and boundedness property.<br>(d) Bolzano-Weierstrass property.<br>(iv) Results such as: Continuous image of compact set is compact, every continuous function from a compact metric space is uniformly continuous. |
| **MODULE II) CONNECTED & PATH-CONNECTED SPACES 15 ECTURES** |
| (i) Separated sets- Definition and examples. Connected and disconnected sets. Connected and disconnected metric spaces.<br>(ii) Results such as: A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected.<br>(iii) Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from $X$ to $\{1,-1\}$ is a constant function.<br>(iv) Path connectedness in $\mathbb{R}^n$, definition and examples.<br>(v) Properties: A path connected subset of $\mathbb{R}^n$ is connected, convex sets are path connected. Connected components. An example of a connected subset of $\mathbb{R}^n$ which is not path connected. |

**Recommanded Book:**


1. S. Kumaresan; Topology of Metric spaces, Narosa Publication Hose, New Delhi, 2nd edition, 2011.


**References Books:**


1. R. R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi., 2020.
2. E. T. Copson, Metric Spaces, Universal Book Stall, New Delhi, 1996.
3. Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis;

3rd Edition, John Wiley and Sons, 2000.

4. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
5. P. K. Jain, K. Ahmed, Metric Spaces, Narosa Publication House, New Delhi, 3rd edition, 1996.
6. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadav; An Introduction to Metric Spaces, Chapman and Hall/CRC press, New York, 1st Edition, 2020.
8. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill education, India, Third Edition, 1976.
9. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill company, Inc., New York, 1st Edition, 1963.

## STRUCTIRE OF THE COURSE

| Semester | Course Code | Course title | No. of Credits | Type | No. of Lectures (Hrs) |
|---|---|---|---|---|---|
| VI | USMTT312 (MAJOR) | Advanced Real Analysis | 2 | Theory | 30 |

**Course Objectives:**

- To introduce students to sequences and series of real-valued functions and develop the concepts of pointwise and uniform convergence through examples and counterexamples.
- To enable students to analyze how uniform convergence influences continuity, integration, and differentiation of functions.
- To develop a strong understanding of power series, including radius and interval of convergence, and the analytical behavior of power series on compact subsets.
- To build problem-solving skills through term-by-term operations on power series and through studying classical functions defined by power series.

**Course Outcomes:** After completion of the course, students will be able to

- define and distinguish pointwise convergence and uniform convergence of sequences and series of functions using suitable examples.
- apply criteria for convergence of series of functions and verify uniform convergence using results such as the Weierstrass M-test.
- examine how uniform convergence affects continuity, differentiation, and integration of the limit function on closed and bounded intervals.
- determine the radius and interval of convergence of power series and analyze convergence on compact subsets.
- represent classical functions using power series and perform term-by-term differentiation and integration of power series.

| **MODULE I) SEQUENCES AND SERIES OF FUNCTIONS 15 LECTURES** |
|---|
| (i) Sequence of real-valued functions: definition and examples. Pointwise convergence of a sequence of functions: definition and examples. Uniform convergence of a sequence of functions: definition, examples. |

| |
|---|
| (ii) Relation between types of convergence: Uniform convergence implies pointwise convergence. Example to show that the converse does not hold.<br>(iii) Series of functions: definition and examples. Convergence of series of functions. Weierstrass M-test (statement only) and illustrative examples.<br>(iv) Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous functions. Statement of conditions under which integration and differentiation of a sequence of functions converge to the integral and derivative of the uniform limit on a closed and bounded interval.<br>(v) Consequences of these properties for series of functions: term-by-term integration and term-by-term differentiation (statements only). |
| **MODULE II) POWER SERIES AND APPLICATIONS**<br>**15 LECTURES** |
| (i) Power series in $\mathbb{R}$ : definition of power series centered at the origin and at a point $a \in \mathbb{R}$. Radius of convergence and region (interval) of convergence, with examples.<br>(ii) Uniform convergence of power series on compact subsets of the interval of convergence and examples.<br>(iii) Term-by-term differentiation and integration of power series (statements only), with examples.<br>(iv) Uniqueness of power series representation. Functions represented by power series and illustrative examples.<br>(v) Classical functions defined by power series such as exponential, cosine, and sine functions. Basic properties of the above functions obtained through their power series expansions. |

**Recommanded Book:**


**1.** Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis; 3rd Edition, John Wiley and Sons, 2000.


**References Books:**


1. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
2. P. K. Jain, K. Ahmed, Metric Spaces, Narosa Publication House, New Delhi, 3rd edition, 1996.
3. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
4. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill

education, India, Third Edition, 1976.
5. S. Kumaresan; Topology of Metric spaces, Narosa Publication House, New Delhi, 2nd edition, 2011.
6. R. R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi., 2020.
7. E. T. Copson, Metric Spaces, Universal Book Stall, New Delhi, 1996.

**EXAMINATION PATTERN FOR**

**USMTT309, USMTT310, USMTT311, USMTT312**

## Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Home Assignment / a quiz on one of the modules. | 05 Marks |
| 03 | Seminar/ group presentation on any one topic related to the syllabus. | 05 Marks |
| | **Total** | **20 Marks** |

**Paper pattern of the Test (Duration: 30 Minutes):**

Q 1: Definitions/MCQ/ Fill in the blanks/True or False with Justification. (04 Marks: 4 x 1).

Q 2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3).

## Semester End Examination (SEE): 30 Marks

| **Duration: 1 Hour** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Attempt Any Two:<br>(d) Question on Module I<br>(b) Question on Module I<br>(c) Question on Module I | 10 Marks |
| Q. 2 A) | Attempt Any Two:<br>(a) Question on Module II<br>(b) Question on Module II<br>(c) Question on Module II | 10 Marks |
| Q. 3 A) | Attempt Any Two:<br>(g) Question on Module I or Module II<br>(h) Question on Module I or Module II<br>(i) Question on Module I or Module II | 10 Marks |
| | **Total** | **30 Marks** |

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | Credits |
|---|---|---|---|---|
| VI (Level 5.5) | USMTP313 | Practical - 305 (Practical based on Introduction to Complex Analysis) | Practical | 02 |

## Course Objectives:

- To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting.
- To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences.
- To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment.
- To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.

## Course Outcomes: After completion of the course, students will be able to

- Apply basic concepts of complex numbers and functions to solve problems and sketch sets in the complex plane.
- Analyze convergence, continuity, and differentiability of complex functions using standard criteria.
- Justify/Check analyticity, harmonicity, and functional properties using appropriate conditions.
- Construct analytic functions, harmonic conjugates, and Möbius transformations.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Limits and Continuity of Functions of Complex Variables |
| **2.** | Derivatives of Functions of Complex Variables |
| **3.** | Analytic Functions |

| | |
|---|---|
| **4.** | Harmonic Function and Harmonic Conjugate |
| **5.** | Complex Mapping |
| **6.** | Mobius Transformation |
| **7.** | Exponential Function and Its Properties |
| **8.** | Trigonometric and Hyperbolic Functions |
| **9.** | Complex Integration |
| **10.** | Cauchy’s Theorem |
| **11.** | Complex Power Series |
| **12.** | Laurent’s Series |
| **13.** | Singularities |
| **14.** | Calculation of Residue |
| **15.** | Cauchy’s Residue Theorem |
| | |
| | **Miscellaneous Theory Questions based on both modules.** |

**Recommanded Book:**


1. J.W. Brown and R.V. Churchill, Complex analysis and Applications, McGraw-Hill, a publication, New York, 9th edition, 2013.


**Reference Books:**


1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable, Orient Blackswan Private Limited - New Delhi, 3rd Edition, 2011.
2. T.W. Gamelin, Complex analysis, Springer Verlag publisher, New York, 2001.
3. R. P. Agarwal, Kanishka Perera, Sandra Pinelas, An introduction to Complex analysis, Springer Verlag publisher, New York, 2011.
4. Mohammad Ibrahim Mir, Complex Analysis: An Introduction, Atlantic Publishers & Distributors (P) Ltd, 2025.

## CREDIT STRUCTURE

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| **VI (Level 5.5)** | USMTP314 | Practical - 306 (Practical based on Ring Theory) | **Practical** | **02** |

## Course Objectives:

- To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting.
- To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences.
- To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment.
- To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Apply fundamental concepts of rings, fields, and their examples.
- Analyze algebraic properties of rings and fields, including substructures and homomorphisms.
- Justify/Check properties of ideals, quotient rings, and field extensions.
- Construct examples of rings, fields, ideals, and ring/field homomorphisms.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Introduction to Rings and Fundamental Properties of Rings |
| **2.** | Subrings of a ring and Integral Domains |
| **3.** | Units and Finite Fields |
| **4.** | Characteristic of Rings |
| **5.** | Ideals of a ring, Quotient Rings and Ideal generated by set of elements |
| **6.** | Prime Ideals |
| **7.** | Maximal Ideals |
| **8.** | Homomorphism of rings, Kernel and Image under ring homomorphism |

| | |
|---|---|
| **9.** | Isomorphism of Rings |
| **10.** | Divisibility of Elements in a Ring, Irreducible & Prime Elements |
| **11.** | Division algorithm in F[X] and GCD of polynomials |
| **12.** | Irreducible polynomials and Eisenstein Criterion for irreducibility |
| **13.** | Euclidean Domains |
| **14.** | Principal Ideal Domains |
| **15.** | Unique Factorization Domains |
| | |
| **Miscellaneous Theory Questions based on both modules.** | |

**Recommanded Book:**


1. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi, 2013.


**References Books:**


1. Michael Artin, Algebra, Pearson Education, India, Second Edition, 2024.
2. J. A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa Publishing House, 2008.
3. S. Adhikari; An Introduction to Commutative Algebra and Number theory, Narosa publication House, New Delhi, India, First Edition, 2001.
4. T. W. Hungerford, Graduate Texts in Mathematics: Algebra, Springer verlag, New York, 4th Edition 2003.
5. I. S. Luther, I. B. S. Passi. Algebra. Vol. II, Narosa publication House, New Delhi, India, 1999.
6. N. S. Gopalkrishnan, University Algebra, New Age International Private Limited, India, 2018.
7. Israel Herstein; Topics in Algebra; Second edition, Wiley Eastern Limited, Second edition, 2006.
8. P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul. Abstract Algebra, second edition, Cambridge University press, 1995.

## CREDIT STRUCTURE

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| **VI (Level 5.5)** | USMTP315 | Practical - 307 (Practical based on Topology of Metric Spaces – II and Advanced Real Analysis) | **Practical** | **02** |

**Course Objectives:**

- To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting.
- To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences.
- To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment.
- To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Apply concepts of compactness, connectedness, and path connectedness in metric spaces, along with techniques involving sequences, series, and power series of functions, to solve theoretical and computational problems.
- Analyze the behavior of sequences and series of functions using convergence tests and topological properties such as compactness and connectedness in metric spaces.
- Justify / Check the validity of convergence, continuity, and topological properties by verifying necessary and sufficient conditions through rigorous mathematical arguments and examples.
- Construct examples and counterexamples of metric spaces, compact and connected sets, and sequences or power series of functions illustrating various convergence and topological phenomena.

# LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Compact sets in $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$. |
| **2.** | Properties of compact sets. |
| **3.** | Equivalent Characterizations of Compactness in $\mathbb{R}$. |
| **4.** | Compactness and Continuity. |
| **5.** | Separated, Connected and Disconnected Sets. |
| **6.** | Connected Sets in $\mathbb{R}$ and Continuous Images. |
| **7.** | Functional Characterization of Connectedness. |
| **8.** | Path Connectedness in $\mathbb{R}^n$ and Related Properties. |
| **9.** | Pointwise and Uniform convergence of a sequence of functions. |
| **10.** | Pointwise and Uniform convergence of a series of functions, Weierstrass M-test. |
| **11.** | Continuity, differentiation, and integration of sequences and series of functions. |
| **12.** | Continuity, differentiation, and integration of series of functions. |
| **13.** | Radius of convergence and interval of convergence of power series. |
| **14.** | Differentiation and integration of power series. |
| **15.** | Properties of exponential, sine and cosine functions through power series. |
| | |
| **Miscellaneous Theory Questions based on both modules.** | |

**Recommanded Books:**


1. S. Kumaresan; Topology of Metric spaces, Narosa Publication Hose, New Delhi, 2nd edition, 2011.
2. Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis; 3rd Edition, John Wiley and Sons, 2000.


**References Books:**


1. R. R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi., 2020.
2. E. T. Copson, Metric Spaces, Universal Book Stall, New Delhi, 1996.
3. Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis; 3rd Edition, John Wiley and Sons, 2000.

4. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
5. P. K. Jain, K. Ahmed, Metric Spaces, Narosa Publication House, New Delhi, 3rd edition, 1996.
6. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
7. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill education, India, Third Edition, 1976.
8. T. M. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
9. P. K. Jain, K. Ahmed, Metric Spaces, Narosa Publication House, New Delhi, 3rd edition, 1996.
10. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
11. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill education, India, Third Edition, 1976.
12. S. Kumaresan; Topology of Metric spaces, Narosa Publication House, New Delhi, 2nd edition, 2011.

**EXAMINATION PATTERN FOR**

**USMTP313, USMTP314, USMTP315**

**Continuous Internal Evaluation (CIE): 20 Marks**

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Overall performance | 05 Marks |
| 03 | Viva | 05 Marks |
| | **Total** | **20 Marks** |

**Semester End Examination (SEE): 30 Marks**

| **Duration: 2 Hours** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Multiple choice questions (Attempt any 5 from 8). | 10 Marks |
| Q. 2 A) | Attempt any Three out of Five. | 15 Marks |
| | **Total** | **25 Marks** |

**Journal: 5 Marks**

**STRUCTURE OF THE COURSE**

| Semester | Course code | Course title | No. of Credits | Type | No. of Lectures |
|---|---|---|---|---|---|
| VI | USMTET316A **(ELECTIVE)** | Graph Theory -II | 2 | Theory | 30 |

## Course Objectives:

- To provide a strong foundation in advanced graph concepts such as Hamiltonian graphs, cube graphs, and line graphs, enabling students to model and analyze relationships in diverse systems.
- To develop proficiency in connectivity and coloring principles, including vertex/edge cuts, chromatic numbers, Brooks' and Vizing's theorems, and chromatic polynomials, for practical problem solving and algorithm design.
- To enhance analytical and problem-solving abilities by studying planar graphs, Euler's formula, non-planarity criteria, dual graphs, Platonic solids, and graph colorability results.
- To apply advanced graph-theoretic concepts-including connectivity, coloring techniques, chromatic polynomials, and planarity principles-to model complex networks, solve theoretical and real-world problems.

## Course Outcomes: After completion of the course, students will be able to

- Understand and recall fundamental concepts of Hamiltonian graphs, cube graphs, line graphs, and graph connectivity, including key theorems such as Ore's, Dirac's, and Whitney's theorems.
- Apply vertex and edge coloring techniques, evaluate chromatic numbers, and utilize Brooks' and Vizing's theorems to solve graph coloring problems.
- Compute and analyze chromatic polynomials and apply their properties to study graph colorability and combinatorial structures.
- Analyze planar graphs, apply Euler's formula, assess non-planarity, study dual graphs and Platonic solids, and understand planar graph colorability including 5-Color and 4-Color theorems.
- Construct and evaluate graph models by integrating connectivity, coloring, chromatic polynomials, and planarity concepts to formulate proofs, design examples and counterexamples, and solve advanced applied problems in graph theory.

| MODULE I) HAMILTONIAN GRAPHS AND CONNECTIVITY 15 LECTURES |
|---|
| (i) Hamiltonian graph, Necessary condition for Hamiltonian graphs using G\S where S is a proper subset of V(G), Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph.<br>(ii) Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.<br>(iii) Vertex and edge cuts, vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs (without proof). |
| **MODULE II) COLORING OF GRAPHS AND PLANAR GRAPHS 15 LECTURES** |
| (i) Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Brooks theorem (Statement only). Edge colouring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Vizing Theorem (Statement only).<br>(ii) Chromatic polynomial of graphs- Recurrence Relation and properties of chromatic polynomials.<br>(iii) Definition of planar graph. Euler formula and its consequences. Non planarity of $K_5$ and $K_{3,3}$ graphs, Dual of a graph. Polyhedron in $R^3$ and existence of exactly five regular polyhedron- (Platonic solids), Colourability of planar graphs- 5- Color theorem for planar graphs (Statement only). |

**Recommended Book:**


1. Douglas B. West, Introduction to Graph Theory, 2nd Edition, Pearson, Education Delhi, India, 2000.


**Reference Books:**


1. M. Behzad, G. Chartrand, Introduction to the Theory of Graphs, Boston, Allyn and Bacon publisher, Boston, 1971.
2. Choudum S. A., A First Course in Graph Theory, Laxmi Publications, India, 2000.

3. Daniel I. A. Cohen, Basic Techniques of Combinatorial Theory, John Wiley & Sons. Publisher, USA, 1st Edition, 1978.
4. R. A. Brualdi; Introductory Combinatorics, Pearson Education, India, 5th Edition, 2019.
5. Santanu Saha Ray, Graph Theory with Algorithms and its Applications in Applied Science and Technology, Springer 2013.
6. J. Clark and D. A. Holtan, A first look of Graph Theory, Allied publishers limited, New Delhi, India, 1st Edition, 1995.
7. J. A. Bondy, U.S.R. Murty, Graph Theory with Applications, MacMillan publisher, London, 1st Edition, 1976.
8. K. Balkrishnan and K. Ranganathan, A Textbook of Graph Theory, Springer publisher, New York, 2nd Edition, 2012.

## STRUCTURE OF THE COURSE

| Semester | Course code | Course title | No. of Credits | Type | No. of Lectures |
|---|---|---|---|---|---|
| VI | USMTET316B **(ELECTIVE)** | Number Theory and Its applications – II | 2 | Theory | 30 |

**Course Objectives:**

- To develop a clear understanding of quadratic residues, Legendre and Jacobi symbols, and the laws of quadratic reciprocity.
- To illustrate the depth and structure of number theory by equipping students with theoretical tools to analyze quadratic congruences and related arithmetic problems.
- To strengthen problem-solving abilities and logical reasoning through rigorous proofs involving reciprocity laws and continued fractions.
- To provide exposure to classical results and foundational ideas in number theory, particularly those arising from continued fractions, that underpin developments in the Mathematical Sciences.

**Course Outcomes:** After completion of the course, students will be able to

- Understand and recall fundamental concepts related to quadratic residues, Legendre and Jacobi symbols, and continued fractions.
- Apply standard theorems and techniques to solve problems involving quadratic congruences, reciprocity laws, and rational approximations of irrational numbers.
- Analyze and interpret properties of quadratic residues, periodic continued fractions, and solutions of classical Diophantine equations like Pell's Equations using appropriate theoretical tools.
- Justify results concerning solvability, uniqueness, or non-solvability of quadratic congruences and Pell's equations through rigorous mathematical proofs.
- Construct and examine illustrative examples and counterexamples related to quadratic reciprocity, continued fractions, and Pell's equations, demonstrating deeper conceptual understanding.

| **MODULE I) QUADRATIC RECIPROCITY** **15 LECTURES** |
|---|
| (i) Quadratic residues and Legendre Symbol, Gauss's Lemma, Theorem on Legendre Symbol $\left(\frac{2}{P}\right)$ and $\left(\frac{3}{P}\right)$ and associated Results. If p is an odd prime |

| | |
|---|---|
| and a is an odd integer with $(a,p) = 1$ then $\left(\frac{2}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} \left[\frac{ka}{p}\right]$. Quadratic Reciprocity law. Theorem on Legendre Symbol $\left(\frac{3}{p}\right)$. (ii) The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli. | |
| **MODULE II) CONTINUED FRACTIONS** | **15 LECTURES** |
| (i) Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence. (ii) Best possible approximations. Periodic continued fractions. Pell's equation $x^2 - dy^2 = 1$, where d is not a square of an integer. Arithmetic functions of number theory: d(n) (or τ(n),) σ(n), $\sigma_k(n)$, ω(n)) and their properties, μ(n) and the Mobius inversion formula. | |

**Recommended Books:**


1. David M. Burton, Elementary Number Theory, Tata McGraw-Hill Edition, India, 7th Edition, 2023.


**Reference Books:**


1. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, The English Language Book Society and Oxford University Press, 1981.
2. Neville Robins, Beginning Number Theory, Jones & Bartlett Learning, USA, 2nd Edition, 2006.
3. S. D. Adhikari, An introduction to Commutative Algebra and Number Theory; Narosa Publishing House, 1999.
4. N. Koblitz; A course in Number theory and Cryptography, Springer publisher, New Delhi, 2nd Edition, 1994.
5. M. Artin; Algebra, Pearson Education publisher, India, 2nd Edition, 2010.
6. K. Ireland, M. Rosen, A classical introduction to Modern Number Theory, Second edition, Springer Verlag India, 1991.
7. I. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc., New York, 5th Edition, 1991.

**EXAMINATION PATTERN FOR**

**USMTET316A, USMTET316B**

## Continuous Internal Evaluation (CIE): 20 Marks

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Home Assignment / a quiz on one of the modules. | 05 Marks |
| 03 | Seminar/ group presentation on any one topic related to the syllabus. | 05 Marks |
| | **Total** | **20 Marks** |

**Paper pattern of the Test (Duration: 30 Minutes):**

Q 1: Definitions/MCQ/ Fill in the blanks/True or False with Justification. (04 Marks: 4 x 1).

Q 2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3).

## Semester End Examination (SEE): 30 Marks

| **Duration: 1 Hour** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Attempt Any Two:<br>(e) Question on Module I<br>(b) Question on Module I<br>(c) Question on Module I | 10 Marks |
| Q. 2 A) | Attempt Any Two:<br>(a) Question on Module II<br>(b) Question on Module II<br>(c) Question on Module II | 10 Marks |
| Q. 3 A) | Attempt Any Two:<br>(j) Question on Module I or Module II<br>(k) Question on Module I or Module II<br>(l) Question on Module I or Module II | 10 Marks |
| | **Total** | **30 Marks** |

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| VI (Level 5.5) | USMTEP317A | Practical - 308 (Practical based on Graph Theory - II) | Practical | 02 |

## Course Objectives:

- To develop the ability to construct Hamiltonian paths and cycles, test necessary and sufficient conditions using Ore's and Dirac's theorems, and compute Hamiltonian closures to analyze complex structural behaviors in graphs.
- To enable students to investigate cube graphs, verify properties such as regularity and bipartiteness, determine Hamiltonian characteristics, construct line graphs, and apply connectivity techniques using vertex and edge cuts.
- To build proficiency in evaluating vertex and edge connectivity, interpret their relationships, and apply classical results such as Whitney's theorem to determine the structural robustness of cubic and connected graphs.
- To cultivate practical competence in graph coloring by computing vertex and edge chromatic numbers, verifying theoretical bounds using Brooks' and Vizing's theorems, and applying chromatic polynomial methods and recurrence relations for colorability analysis.
- To strengthen students' analytical and modelling skills by performing planarity tests, applying Euler's formula, demonstrating non-planarity, constructing dual graphs and Platonic solid representations.

## Course Outcomes: After completion of the course, students will be able to

- Construct and analyze Hamiltonian paths, Hamiltonian cycles, cube graphs, line graphs, and related structures by applying necessary and sufficient conditions including Ore's and Dirac's theorems to evaluate Hamiltonicity, connectivity, and core structural properties of graphs.
- Compute and interpret vertex cuts, edge cuts, and both forms of connectivity; examine their interrelationships; and verify classical structural results such as the connectivity of cubic graphs and Whitney's theorem on 2-vertex connectivity.
- Implement vertex and edge coloring methods for standard graphs, determine chromatic numbers, verify theoretical bounds using Brooks' and Vizing's theorems,

and compute chromatic polynomials using recurrence relations and associated properties.

- Analyze planar and non-planar graphs through planarity testing and Euler's formula, construct dual graphs and Platonic solid representations.

## LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Hamiltonian path, cycle, graph and testing Necessary Conditions |
| **2.** | Ore's theorem, Dirac's theorem and its properties |
| **3.** | Hamiltonian Closure of Graphs. |
| **4.** | Cube graphs and properties like regular, bipartite, connected and Hamiltonian nature of cube graph. |
| **5.** | Characterization of Line Graphs. |
| **6.** | Vertex cuts, edge cuts and Relation Between Vertex and Edge Connectivity. |
| **7.** | Connectivity of Cubic Graphs, Whitney's theorem. |
| **8.** | Vertex Coloring of graphs, Brooks Theorem. |
| **9.** | Edge Coloring of graphs, Vizing's Theorem. |
| **10.** | Chromatic Polynomial of Graphs. |
| **11.** | Chromatic Polynomial of graphs and its recursive formula. |
| **12.** | Planarity testing, Euler's Formula. |
| **13.** | Non-planarity testing and its characteristics. |
| **14.** | Dual of a graph. |
| **15.** | Platonic Solids and Graph Representation. |

**Recommended Book:**


1. Douglas B. West, Introduction to Graph Theory, 2nd Edition, Pearson, Education Delhi, India, 2000.


**Reference Books:**


1. M. Behzad, G. Chartrand, Introduction to the Theory of Graphs, Boston, Allyn and Bacon publisher, Boston, 1971.
2. Choudum S. A., A First Course in Graph Theory, Laxmi Publications, India, 2000.
3. Daniel I. A. Cohen, Basic Techniques of Combinatorial Theory, John Wiley &

Sons. Publisher, USA, 1st Edition, 1978.
4. R. A. Brualdi; Introductory Combinatorics, Pearson Education, India, 5th Edition, 2019.
5. Santanu Saha Ray, Graph Theory with Algorithms and its Applications in Applied Science and Technology, Springer 2013.
6. J. Clark and D. A. Holtan, A first look of Graph Theory, Allied publishers limited, New Delhi, India, 1st Edition, 1995.
7. J. A. Bondy, U.S.R. Murty, Graph Theory with Applications, MacMillan publisher, London, 1st Edition, 1976.
8. K. Balkrishnan and K. Ranganathan, A Textbook of Graph Theory, Springer publisher, New York, 2nd Edition, 2012.

**CREDIT STRUCTURE**

| Semester | Course Code | Title of the Course | Category of Course | No. of Credits |
|---|---|---|---|---|
| VI (Level 5.5) | USMTEP317B | Practical - 309 (Practical based on Number Theory and Its applications - II) | Practical | 02 |

## Course Objectives:

- To develop a clear understanding of quadratic residues, Legendre and Jacobi symbols, and the laws of quadratic reciprocity.
- To illustrate the depth and structure of number theory by equipping students with theoretical tools to analyze quadratic congruences and related arithmetic problems.
- To strengthen problem-solving abilities and logical reasoning through rigorous proofs involving reciprocity laws and continued fractions.
- To provide exposure to classical results and foundational ideas in number theory, particularly those arising from continued fractions, that underpin developments in the Mathematical Sciences.

## Course Outcomes: After completion of the course, students will be able to

- Apply standard theorems and techniques to solve problems involving quadratic congruences, reciprocity laws, and rational approximations of irrational numbers.
- Analyze and interpret properties of quadratic residues, periodic continued fractions, and solutions of classical Diophantine equations like Pell's Equations using appropriate theoretical tools.
- Justify results concerning solvability, uniqueness, or non-solvability of quadratic congruences and Pell's equations through rigorous mathematical proofs.
- Construct and examine illustrative examples and counterexamples related to quadratic reciprocity, continued fractions, and Pell's equations, demonstrating deeper conceptual understanding.

# LIST OF PRACTICALS

| | |
|---|---|
| **1.** | Solving general quadratic congruences |
| **2.** | Quadratic Residues and Euler's Criterion. |
| **3.** | Properties of Legendre Symbol. |
| **4.** | Gauss' Lemma. |
| **5.** | Law of Quadratic Reciprocity |
| **6.** | Quadratic congruences modulo power of odd prime. |
| **7.** | Quadratic congruences modulo power of 2. |
| **8.** | Quadratic congruences modulo any composite number. |
| **9.** | Writing a rational number as a continued fraction and a finite continued fraction as a rational number. |
| **10.** | Convergents of a finite continued fraction and properties. |
| **11.** | Solving a Diophantine equation using continued fraction |
| **12.** | Finding the irrational numbers associated to infinite continued fractions |
| **13.** | Finding the infinite continued fractions of the irrational numbers |
| **14.** | Rational approximations to irrational numbers and order of convergence, Best possible approximations. |
| **15.** | Solving Pell's equation using infinite continued fractions. |
| | |
| **Miscellaneous theory questions based on above two modules.** | |

**Recommended Books:**

1. David M. Burton, Elementary Number Theory, Tata McGraw-Hill Edition, India, 7th Edition, 2023.

**Reference Books:**

1. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, The English Language Book Society and Oxford University Press, 1981.
2. Neville Robins, Beginning Number Theory, Jones & Bartlett Learning, USA, 2nd Edition, 2006.
3. S. D. Adhikari, An introduction to Commutative Algebra and Number Theory;

Narosa Publishing House, 1999.
4. N. Koblitz; A course in Number theory and Cryptography, Springer publisher, New Delhi, 2nd Edition, 1994.
5. M. Artin; Algebra, Pearson Education publisher, India, 2nd Edition, 2010.
6. K. Ireland, M. Rosen, A classical introduction to Modern Number Theory, Second edition, Springer Verlag India, 1991.
7. I. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc., New York, 5th Edition, 1991.

**EXAMINATION PATTERN FOR**
**USMTEP317A, USMTEP317B**

**Continuous Internal Evaluation (CIE): 20 Marks**

| Sr. No. | Particulars | Marks |
|---|---|---|
| 01 | One Class Test | 10 Marks |
| 02 | Overall performance | 05 Marks |
| 03 | Viva | 05 Marks |
| | **Total** | **20 Marks** |

**Semester End Examination (SEE): 30 Marks**

| **Duration: 2 Hours** | | **Marks: 30** |
|---|---|---|
| Q. 1 A) | Multiple choice questions (Attempt any 5 from 8). | 10 Marks |
| Q. 2 A) | Attempt any Three out of Five. | 15 Marks |
| | **Total** | **25 Marks** |

**Journal: 5 Marks**